ERRATUM TO: DEFORMATION QUANTIZATION IN ALGEBRAIC GEOMETRY

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Abstract. This note contains a correction of the *proofs* of the main results of the paper [A. Yekutieli, Deformation quantization in algebraic geometry, Adv. Math. **198** (2005), 383-432]. The results are correct as originally stated.

0. Introduction

This note contains a correction of the *proofs* of the main results of [Ye1], namely Theorems 0.1 and 0.2. The results are correct as originally stated.

The mistake in my original proofs was discovered Michel Van den Bergh, and I thank him for calling my attention to it. The way to fix the proofs is essentially contained in his paper [VdB].

Let me begin by explaining the mistake. As can be seen in Example 0.1 below, the mistake itself is of a rather elementary nature, but it was obscured by the complicated context.

Suppose $\mathbb{K}$ is a field of characteristic 0, and X is a smooth separated n-dimensional scheme over $\mathbb{K}$. Recall that the coordinate bundle $\operatorname{Coor} X$ is an infinite dimensional bundle over X, with free action by the group $\mathrm{GL}_{n,\mathbb{K}}$. The quotient bundle is by definition

$$\operatorname{LCC} X := \operatorname{Coor} X / \mathrm{GL}_{n,\mathbb{K}},$$

and the projection $\pi_{\mathrm{gl}} : \operatorname{Coor} X \to \operatorname{LCC} X$ is a $\mathrm{GL}_{n,\mathbb{K}}$-torsor.

The erroneous (implicit) assertion in [Ye1] is that the de Rham complexes satisfy

$$(\pi_{\mathrm{gl}*}\, \Omega_{\operatorname{Coor} X})^{\mathrm{GL}_n(\mathbb{K})} = \Omega_{\operatorname{LCC} X}.$$

From that I deduced (incorrectly, top of page 424) that the Maurer-Cartan form ω_{MC} is a global section of the sheaf

$$\Omega^1_{\operatorname{LCC} X} \,\widehat{\otimes}_{\mathcal{O}_{\operatorname{LCC} X}} \pi^{\widehat{*}}_{\mathrm{lcc}}(\mathcal{P}_X \otimes_{\mathcal{O}_X} \mathcal{T}^0_{\mathrm{poly},X}).$$

(This false, as can be seen from [VdB, Lemma 6.5.1]). This led to many incorrect formulas in [Ye1, Section 7].

The correct thing to do is to work with the infinitesimal action of the Lie algebra $\mathfrak{g} := \mathfrak{gl}_n(\mathbb{K})$. For $v \in \mathfrak{g}$ one has the contraction (inner derivative) ι_v, which is a degree -1 derivation of the de Rham complex $\pi_{\mathrm{gl}*}\, \Omega_{\operatorname{Coor} X}$. Recall that the Lie derivative is $\mathrm{L}_v := \mathrm{d} \circ \iota_v + \iota_v \circ \mathrm{d}$. A local section $\omega \in \pi_{\mathrm{gl}*}\, \Omega_{\operatorname{Coor} X}$ is said to be $\mathfrak{g}$-invariant if $\iota_v(\omega) = \mathrm{L}_v(\omega) = 0$ for all $v \in \mathfrak{g}$. According to [VdB, Lemma 9.2.3] one has

$$(\pi_{\mathrm{gl}*}\, \Omega_{\operatorname{Coor} X})^{\mathfrak{g}} = \Omega_{\operatorname{LCC} X}.$$

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It is worthwhile to note that in my incorrect proof there was no need to invoke Kontsevich's property (P5) from [Ko]. The correct proof does require property (P5) – cf. [VdB, Lemma 9.2.1].

Example 0.1. Here is a simplified example. Suppose G is the affine algebraic group $\mathrm{GL}_{1,\mathbb{K}} = \operatorname{Spec} \mathbb{K}[t, t^{-1}]$, and X is the variety G, with regular left action. The group of rational points is $G(\mathbb{K}) = \mathbb{K}^{\times}$. The action of G on X is free, the invariant ring is $\mathcal{O}(X)^{G(\mathbb{K})} = \mathbb{K}$, and the quotient is $X/G = \operatorname{Spec} \mathbb{K}$. For the de Rham complex

$$\Omega(X) = \mathcal{O}(X) \oplus \Omega^1(X) = \mathbb{K}[t, t^{-1}] \oplus \mathbb{K}[t, t^{-1}] \cdot \mathrm{d}t$$

we have $\Omega(X)^{G(\mathbb{K})} \neq \mathbb{K}$, since it contains $t^{-1}\mathrm{d}t$. But for the infinitesimal action of the Lie algebra $\mathfrak{g} := \mathfrak{gl}_1(\mathbb{K})$ it is easy to see that $\Omega(X)^{\mathfrak{g}} = \mathbb{K}$.

After some deliberation I decided that the best way to present the erratum is by completely rewriting [Ye1, Section 7]. This is Section 1 below. Section 2 contains some additional minor corrections to [Ye1].

1. The Global L_∞ Quasi-isomorphism

This is a revised version of [Ye1, Section 7]. In this section we prove the main results of the paper [Ye1], namely Theorem 0.1 (which is repeated here as Corollary 1.19), and Theorem 0.2 (which is repeated here, with more details, as Theorem 1.2). Throughout $\mathbb{K}$ is a field containing $\mathbb{R}$, and X is a smooth irreducible separated n-dimensional scheme over $\mathbb{K}$. We use all notation, definitions and results of [Ye1, Sections 1-6] freely. However the bibliography references relate to the list at the end of this note.

Suppose $\boldsymbol{U} = \{U_0, \ldots, U_m\}$ is an open covering of the scheme X, consisting of affine open sets, each admitting an étale coordinate system, namely an étale morphism $U_i \to \mathbf{A}^n_{\mathbb{K}}$. For every i let $\sigma_i : U_i \to \operatorname{LCC} X$ be the corresponding section of $\pi_{\mathrm{lcc}} : \operatorname{LCC} X \to X$, and let $\boldsymbol{\sigma}$ be the resulting simplicial section (see [Ye1, Theorem 6.5]).

Let $\mathcal{M}$ be a bounded below complex of quasi-coherent $\mathcal{O}_X$-modules. The mixed resolution $\operatorname{Mix}_{\boldsymbol{U}}(\mathcal{M})$ was defined in [Ye1, Section 6]. For any integer i let

$$\mathrm{G}^i \operatorname{Mix}_{\boldsymbol{U}}(\mathcal{M}) := \bigoplus\nolimits_{j=i}^{\infty} \operatorname{Mix}^j_{\boldsymbol{U}}(\mathcal{M}),$$

so $\{\mathrm{G}^i \operatorname{Mix}_{\boldsymbol{U}}(\mathcal{M})\}_{i \in \mathbb{Z}}$ is a descending filtration of $\operatorname{Mix}_{\boldsymbol{U}}(\mathcal{M})$ by subcomplexes, with $\mathrm{G}^i \operatorname{Mix}_{\boldsymbol{U}}(\mathcal{M}) = \operatorname{Mix}_{\boldsymbol{U}}(\mathcal{M})$ for $i \leq 0$, and $\bigcap_i \mathrm{G}^i \operatorname{Mix}_{\boldsymbol{U}}(\mathcal{M}) = 0$. Let

$$\operatorname{gr}^i_{\mathrm{G}} \operatorname{Mix}_{\boldsymbol{U}}(\mathcal{M}) := \mathrm{G}^i \operatorname{Mix}_{\boldsymbol{U}}(\mathcal{M}) \ / \ \mathrm{G}^{i+1} \operatorname{Mix}_{\boldsymbol{U}}(\mathcal{M})$$

and $\operatorname{gr}_{\mathrm{G}} \operatorname{Mix}_{\boldsymbol{U}}(\mathcal{M}) := \bigoplus_i \operatorname{gr}^i_{\mathrm{G}} \operatorname{Mix}_{\boldsymbol{U}}(\mathcal{M})$.

By [Ye1, Proposition 6.3], if $\mathcal{G}_X$ is either $\mathcal{T}_{\mathrm{poly},X}$ or $\mathcal{D}_{\mathrm{poly},X}$, then $\operatorname{Mix}_{\boldsymbol{U}}(\mathcal{G}_X)$ is a sheaf of DG Lie algebras on X, and the inclusion

$$\eta_{\mathcal{G}} : \mathcal{G}_X \to \operatorname{Mix}_{\boldsymbol{U}}(\mathcal{G}_X)$$

is a DG Lie algebra quasi-isomorphism.

Note that if $\phi : \operatorname{Mix}_{\boldsymbol{U}}(\mathcal{M}) \to \operatorname{Mix}_{\boldsymbol{U}}(\mathcal{N})$ is a homomorphism of complexes that respects the filtration $\{\mathrm{G}^i \operatorname{Mix}_{\boldsymbol{U}}\}$, then there exists an induced homomorphism of complexes

$$\operatorname{gr}_{\mathrm{G}}(\phi) : \operatorname{gr}_{\mathrm{G}} \operatorname{Mix}_{\boldsymbol{U}}(\mathcal{M}) \to \operatorname{gr}_{\mathrm{G}} \operatorname{Mix}_{\boldsymbol{U}}(\mathcal{N}).$$

Suppose $\mathcal{G}$ and $\mathcal{H}$ are sheaves of DG Lie algebras on a topological space Y. An L_∞ morphism $\Psi : \mathcal{G} \to \mathcal{H}$ is a sequence of sheaf morphisms $\psi_j : \prod^j \mathcal{G} \to \mathcal{H}$, such that for every open set $V \subset Y$ the sequence $\{\Gamma(V, \psi_j)\}_{j \geq 1}$ is an L_∞ morphism $\Gamma(V, \mathcal{G}) \to \Gamma(V, \mathcal{H})$. If $\psi_1 : \mathcal{G} \to \mathcal{H}$ is a quasi-isomorphism then Ψ is called an L_∞ quasi-isomorphism.

Recall that there is a canonical quasi-isomorphism of complexes of $\mathcal{O}_X$-modules

$$\mathcal{U}_1 : \mathcal{T}_{\mathrm{poly},X} \to \mathcal{D}_{\mathrm{poly},X}. \tag{1.1}$$

According to [Ye2, Theorem 4.17], the induced homomorphism

$$\mathrm{gr}_{\mathrm{G}}(\mathrm{Mix}_{\boldsymbol{U}}(\mathcal{U}_1)) : \mathrm{gr}_{\mathrm{G}} \mathrm{Mix}_{\boldsymbol{U}}(\mathcal{T}_{\mathrm{poly},X}) \to \mathrm{gr}_{\mathrm{G}} \mathrm{Mix}_{\boldsymbol{U}}(\mathcal{D}_{\mathrm{poly},X})$$

is a quasi-isomorphism.

Theorem 1.2. *Let X be an irreducible smooth separated $\mathbb{K}$-scheme. Let $\boldsymbol{U} = \{U_0, \ldots, U_m\}$ be an open covering of X consisting of affine open sets, each admitting an étale coordinate system, and let $\boldsymbol{\sigma}$ be the associated simplicial section of the bundle $\mathrm{LCC}\, X \to X$. Then there is an induced L_∞ quasi-isomorphism*

$$\Psi_{\boldsymbol{\sigma}} = \{\Psi_{\boldsymbol{\sigma};j}\}_{j \geq 1} : \mathrm{Mix}_{\boldsymbol{U}}(\mathcal{T}_{\mathrm{poly},X}) \to \mathrm{Mix}_{\boldsymbol{U}}(\mathcal{D}_{\mathrm{poly},X}).$$

The homomorphism $\Psi_{\boldsymbol{\sigma};1}$ respects the filtration $\{\mathrm{G}^i \mathrm{Mix}_{\boldsymbol{U}}\}$, and

$$\mathrm{gr}_{\mathrm{G}}(\Psi_{\boldsymbol{\sigma};1}) = \mathrm{gr}_{\mathrm{G}}(\mathrm{Mix}_{\boldsymbol{U}}(\mathcal{U}_1)).$$

Proof. Let Y be some $\mathbb{K}$-scheme, and denote by $\mathbb{K}_Y$ the constant sheaf. For any p we view Ω^p_Y as a discrete inv $\mathbb{K}_Y$-module, and we put on $\Omega_Y = \bigoplus_{p \in \mathbb{N}} \Omega^p_Y$ direct sum dir-inv structure. So Ω_Y is a discrete (and hence complete) DG algebra in $\mathsf{Dir\,Inv\,Mod}\, \mathbb{K}_Y$.

We shall abbreviate $\mathcal{A} := \Omega_{\mathrm{Coor}\, X}$, so that $\mathcal{A}^0 = \mathcal{O}_{\mathrm{Coor}\, X}$ etc. As explained above, $\mathcal{A}$ is a DG algebra in $\mathsf{Dir\,Inv\,Mod}\, \mathbb{K}_{\mathrm{Coor}\, X}$, with discrete (but not trivial) dir-inv module structure.

There are sheaves of DG Lie algebras $\mathcal{A} \mathbin{\widehat{\otimes}} \mathcal{T}_{\mathrm{poly}}(\mathbb{K}[[\boldsymbol{t}]])$ and $\mathcal{A} \mathbin{\widehat{\otimes}} \mathcal{D}_{\mathrm{poly}}(\mathbb{K}[[\boldsymbol{t}]])$ on the scheme $\mathrm{Coor}\, X$. The differentials are $\mathrm{d}_{\mathrm{for}} = \mathrm{d} \otimes \mathbf{1}$ and $\mathrm{d}_{\mathrm{for}} + \mathbf{1} \otimes \mathrm{d}_{\mathcal{D}}$ respectively. As explained just prior to [Ye1, Theorem 3.16], $\mathcal{U}$ extends to a continuous $\mathcal{A}$-multilinear L_∞ morphism

$$\mathcal{U}_{\mathcal{A}} = \{\mathcal{U}_{\mathcal{A};j}\}_{j \geq 1} : \mathcal{A} \mathbin{\widehat{\otimes}} \mathcal{T}_{\mathrm{poly}}(\mathbb{K}[[\boldsymbol{t}]]) \to \mathcal{A} \mathbin{\widehat{\otimes}} \mathcal{D}_{\mathrm{poly}}(\mathbb{K}[[\boldsymbol{t}]])$$

of sheaves of DG Lie algebras on $\mathrm{Coor}\, X$.

The MC form $\omega := \omega_{\mathrm{MC}}$ is a global section of $\mathcal{A}^1 \mathbin{\widehat{\otimes}} \mathcal{T}^0_{\mathrm{poly}}(\mathbb{K}[[\boldsymbol{t}]])$ satisfying the MC equation in the DG Lie algebra $\mathcal{A} \mathbin{\widehat{\otimes}} \mathcal{T}_{\mathrm{poly}}(\mathbb{K}[[\boldsymbol{t}]])$. See [Ye1, Proposition 5.9]. According to [Ye1, Theorem 3.16], the global section $\omega' := \mathcal{U}_{\mathcal{A};1}(\omega) \in \mathcal{A}^1 \mathbin{\widehat{\otimes}} \mathcal{D}^0_{\mathrm{poly}}(\mathbb{K}[[\boldsymbol{t}]])$ is a solution of the MC equation in the DG Lie algebra $\mathcal{A} \mathbin{\widehat{\otimes}} \mathcal{D}_{\mathrm{poly}}(\mathbb{K}[[\boldsymbol{t}]])$, and there is a continuous $\mathcal{A}$-multilinear L_∞ morphism

$$\mathcal{U}_{\mathcal{A},\omega} = \{\mathcal{U}_{\mathcal{A},\omega;j}\}_{j \geq 1} : \big(\mathcal{A} \mathbin{\widehat{\otimes}} \mathcal{T}_{\mathrm{poly}}(\mathbb{K}[[\boldsymbol{t}]])\big)_{\omega} \to \big(\mathcal{A} \mathbin{\widehat{\otimes}} \mathcal{D}_{\mathrm{poly}}(\mathbb{K}[[\boldsymbol{t}]])\big)_{\omega'}$$

between the twisted DG Lie algebras. The formula is

$$\mathcal{U}_{\mathcal{A},\omega;j}(\gamma_1 \cdots \gamma_j) = \sum_{k \geq 0} \tfrac{1}{(j+k)!} \mathcal{U}_{\mathcal{A};j+k}(\omega^k \cdot \gamma_1 \cdots \gamma_j) \tag{1.3}$$

for $\gamma_1, \ldots, \gamma_j \in \mathcal{A} \mathbin{\widehat{\otimes}} \mathcal{T}_{\mathrm{poly}}(\mathbb{K}[[\boldsymbol{t}]])$. The two twisted DG Lie algebras have differentials $\mathrm{d}_{\mathrm{for}} + \mathrm{ad}(\omega)$ and $\mathrm{d}_{\mathrm{for}} + \mathrm{ad}(\omega') + \mathbf{1} \otimes \mathrm{d}_{\mathcal{D}}$ respectively.

This sum in (1.3) is actually finite, the number of nonzero terms in it depending on the bidegree of $\gamma_1 \cdots \gamma_j$. Indeed, if $\gamma_1 \cdots \gamma_j \in \mathcal{A}^q \mathbin{\widehat{\otimes}} \mathcal{T}^p_{\mathrm{poly}}(\mathbb{K}[[\boldsymbol{t}]])$, then

$$\mathcal{U}_{\mathcal{A};j+k}(\omega^k \cdot \gamma_1 \cdots \gamma_j) \in \mathcal{A}^{q+k} \mathbin{\widehat{\otimes}} \mathcal{D}^{p+1-j-k}_{\mathrm{poly}}(\mathbb{K}[[\boldsymbol{t}]]), \tag{1.4}$$

which is is zero for $k > p - j + 2$; see proof of [Ye2, Theorem 3.23].

By [Ye1, Theorem 5.6] (the universal Taylor expansions) there are canonical isomorphisms of graded Lie algebras in $\mathsf{Dir\,Inv\,Mod}\,\mathbb{K}_{\mathrm{Coor}\,X}$

$$\mathcal{A} \mathbin{\widehat{\otimes}} \mathcal{T}_{\mathrm{poly}}(\mathbb{K}[[\boldsymbol{t}]]) \cong \mathcal{A} \mathbin{\widehat{\otimes}}_{\mathcal{A}^0} \pi^{\widehat{*}}_{\mathrm{coor}}(\mathcal{P}_X \otimes_{\mathcal{O}_X} \mathcal{T}_{\mathrm{poly},X}) \tag{1.5}$$

and

$$\mathcal{A} \mathbin{\widehat{\otimes}} \mathcal{D}_{\mathrm{poly}}(\mathbb{K}[[\boldsymbol{t}]]) \cong \mathcal{A} \mathbin{\widehat{\otimes}}_{\mathcal{A}^0} \pi^{\widehat{*}}_{\mathrm{coor}}(\mathcal{P}_X \otimes_{\mathcal{O}_X} \mathcal{D}_{\mathrm{poly},X}). \tag{1.6}$$

[Ye1, Proposition 5.8] tells us that

$$\mathrm{d}_{\mathrm{for}} + \mathrm{ad}(\omega) = \nabla_{\mathcal{P}}$$

under these identifications. Therefore

$$\begin{aligned} \mathcal{U}_{\mathcal{A},\omega} = \{\mathcal{U}_{\mathcal{A},\omega;j}\}_{j \geq 1} : \mathcal{A} \mathbin{\widehat{\otimes}}_{\mathcal{A}^0} \pi^{\widehat{*}}_{\mathrm{coor}}(\mathcal{P}_X \otimes_{\mathcal{O}_X} \mathcal{T}_{\mathrm{poly},X}) \\ \to \mathcal{A} \mathbin{\widehat{\otimes}}_{\mathcal{A}^0} \pi^{\widehat{*}}_{\mathrm{coor}}(\mathcal{P}_X \otimes_{\mathcal{O}_X} \mathcal{D}_{\mathrm{poly},X}) \end{aligned} \tag{1.7}$$

is a continuous $\mathcal{A}$-multilinear L_∞ morphism between these DG Lie algebras, whose differentials are $\nabla_{\mathcal{P}}$ and $\nabla_{\mathcal{P}} + \mathbf{1} \otimes \mathrm{d}_{\mathcal{D}}$ respectively. As in the proof of [Ye1, Theorem 5.6], under the identifications (1.5) and (1.6) we have the equality

$$\mathcal{U}_{\mathcal{A};1} = \mathbf{1} \otimes \pi^{\widehat{*}}_{\mathrm{coor}}(\mathbf{1} \otimes \mathcal{U}_1), \tag{1.8}$$

i.e. it is the pullback of the map (1.1).

Let us filter the DG algebra $\mathcal{A}$ by the descending filtration $\{\mathrm{G}^i \mathcal{A}\}_{i \in \mathbb{Z}}$, where $\mathrm{G}^i \mathcal{A} := \bigoplus_{j=i}^{\infty} \mathcal{A}^i$. The DG Lie algebras appearing in equation (1.7) inherit this filtration. From formulas (1.3) and (1.4) we see that the homomorphism of complexes $\mathcal{U}_{\mathcal{A},\omega;1}$ respects the filtration, and from (1.8) we see that

$$\mathrm{gr}_{\mathrm{G}}(\mathcal{U}_{\mathcal{A},\omega;1}) = \mathrm{gr}_{\mathrm{G}}(\mathcal{U}_{\mathcal{A};1}) = \mathbf{1} \otimes \pi^{\widehat{*}}_{\mathrm{coor}}(\mathbf{1} \otimes \mathcal{U}_1).$$

Let $n := \dim X$. As noted earlier, the action of $\mathfrak{g} := \mathfrak{gl}_n(\mathbb{K})$ gives

$$(\pi_{\mathrm{gl}*}\, \mathcal{A})^{\mathfrak{g}} = (\pi_{\mathrm{gl}*}\, \Omega_{\mathrm{Coor}\,X})^{\mathfrak{g}} = \Omega_{\mathrm{LCC}\,X}.$$

According to [VdB, Lemma 9.2.1], the L_∞ morphism $\mathcal{U}_{\mathcal{A},\omega}$ commutes with the action of the Lie algebra $\mathfrak{g}$. Therefore $\mathcal{U}_{\mathcal{A},\omega}$ descends (i.e. restricts) to a continuous $\Omega_{\mathrm{LCC}\,X}$-multilinear L_∞ morphism

$$\begin{aligned} \mathcal{U}^{\mathfrak{g}}_{\mathcal{A},\omega} : \Omega_{\mathrm{LCC}\,X} \mathbin{\widehat{\otimes}}_{\mathcal{O}_{\mathrm{LCC}\,X}} \pi^{\widehat{*}}_{\mathrm{lcc}}(\mathcal{P}_X \otimes_{\mathcal{O}_X} \mathcal{T}_{\mathrm{poly},X}) \\ \to\ \Omega_{\mathrm{LCC}\,X} \mathbin{\widehat{\otimes}}_{\mathcal{O}_{\mathrm{LCC}\,X}} \pi^{\widehat{*}}_{\mathrm{lcc}}(\mathcal{P}_X \otimes_{\mathcal{O}_X} \mathcal{D}_{\mathrm{poly},X}). \end{aligned} \tag{1.9}$$

The DG Lie algebras in formula (1.9) also have filtrations $\{\mathrm{G}^j\}_{j \in \mathbb{Z}}$, the homomorphism $\mathcal{U}^{\mathfrak{g}}_{\mathcal{A},\omega;1}$ respects this filtration, and we now have

$$\mathrm{gr}_{\mathrm{G}}(\mathcal{U}^{\mathfrak{g}}_{\mathcal{A},\omega;1}) = \mathrm{gr}_{\mathrm{G}}(\mathcal{U}^{\mathfrak{g}}_{\mathcal{A};1}) = \mathbf{1} \otimes \pi^{\widehat{*}}_{\mathrm{lcc}}(\mathbf{1} \otimes \mathcal{U}_1). \tag{1.10}$$

According to [Ye1, Theorem 6.4] there are induced operators

$$\Psi_{\boldsymbol{\sigma};j} := \boldsymbol{\sigma}^*(\mathcal{U}^{\mathfrak{g}}_{\mathcal{A},\omega;j}) : \prod\nolimits^j \mathrm{Mix}_{\boldsymbol{U}}(\mathcal{T}_{\mathrm{poly},X}) \to \mathrm{Mix}_{\boldsymbol{U}}(\mathcal{D}_{\mathrm{poly},X})$$

for $j \geq 1$. The L_∞ identities in [Ye1, Definition 3.7], when applied to the L_∞ morphism $\mathcal{U}^{\mathfrak{g}}_{\mathcal{A},\omega}$, are of the form considered in [Ye1, Theorem 6.4(iii)]. Therefore these

identities are preserved by $\boldsymbol{\sigma}^*$, and we conclude that the sequence $\Psi_{\boldsymbol{\sigma}} = \{\Psi_{\boldsymbol{\sigma},j}\}_{j=1}^{\infty}$ is an L_∞ morphism. Furthermore, $\Psi_{\boldsymbol{\sigma};1}$ respects the filtration $\{\mathrm{G}^i \operatorname{Mix}_{\boldsymbol{U}}\}$, and from (1.10) we get

$$\operatorname{gr}_{\mathrm{G}}(\Psi_{\boldsymbol{\sigma};1}) = \operatorname{gr}_{\mathrm{G}}(\boldsymbol{\sigma}^*(\mathcal{U}^{\mathfrak{g}}_{\mathcal{A};1})) = \operatorname{gr}_{\mathrm{G}}(\operatorname{Mix}_{\boldsymbol{U}}(\mathcal{U}_1)). \tag{1.11}$$

According to [Ye2, Theorem 4.17] the homomorphism $\operatorname{gr}_{\mathrm{G}}(\operatorname{Mix}_{\boldsymbol{U}}(\mathcal{U}_1))$ is a quasi-isomorphism. Since the complexes $\operatorname{Mix}_{\boldsymbol{U}}(\mathcal{T}_{\mathrm{poly},X})$ and $\operatorname{Mix}_{\boldsymbol{U}}(\mathcal{D}_{\mathrm{poly},X})$ are bounded below, and the filtration is nonnegative and exhaustive, it follows that $\Psi_{\boldsymbol{\sigma};1}$ is also a quasi-isomorphism. □

Corollary 1.12. *Taking global sections in Theorem* 1.2 *we get an* L_∞ *quasi-isomorphism*

$$\Gamma(X, \Psi_{\boldsymbol{\sigma}}) = \{\Gamma(X, \Psi_{\boldsymbol{\sigma};j})\}_{j\geq 1} : \Gamma\big(X, \operatorname{Mix}_{\boldsymbol{U}}(\mathcal{T}_{\mathrm{poly},X})\big) \to \Gamma\big(X, \operatorname{Mix}_{\boldsymbol{U}}(\mathcal{D}_{\mathrm{poly},X})\big).$$

Proof. Theorem 1.2 tells us that $\Psi_{\boldsymbol{\sigma};1}$ is a quasi-isomorphisms of complexes of sheaves. By [Ye1, Theorem 6.2] it follows that

$$\Gamma(X, \Psi_{\boldsymbol{\sigma};1}) : \Gamma\big(X, \operatorname{Mix}_{\boldsymbol{U}}(\mathcal{T}_{\mathrm{poly},X})\big) \to \Gamma\big(X, \operatorname{Mix}_{\boldsymbol{U}}(\mathcal{D}_{\mathrm{poly},X})\big)$$

is a quasi-isomorphism. □

Corollary 1.13. *The data* $(\boldsymbol{U}, \boldsymbol{\sigma})$ *induces a bijection*

$$\begin{aligned} \operatorname{MC}(\Psi_{\boldsymbol{\sigma}}) :\ & \operatorname{MC}\Big(\Gamma\big(X, \operatorname{Mix}_{\boldsymbol{U}}(\mathcal{T}_{\mathrm{poly},X})\big)[[\hbar]]^+\Big) \\ & \xrightarrow{\simeq} \operatorname{MC}\Big(\Gamma\big(X, \operatorname{Mix}_{\boldsymbol{U}}(\mathcal{D}_{\mathrm{poly},X})\big)[[\hbar]]^+\Big). \end{aligned}$$

Proof. Use Corollary 1.12 and [Ye1, Corollary 3.10]. □

Recall that $\mathcal{T}_{\mathrm{poly}}(X) = \Gamma(X, \mathcal{T}_{\mathrm{poly},X})$ and $\mathcal{D}^{\mathrm{nor}}_{\mathrm{poly}}(X) = \Gamma(X, \mathcal{D}^{\mathrm{nor}}_{\mathrm{poly},X})$; and the latter is the DG Lie algebra of global poly differential operators that vanish if one of their arguments is 1.

Suppose $f : X' \to X$ is an étale morphism. According to [Ye2, Prposition 4.6] there are DG Lie algebra homomorphisms $f^* : \mathcal{T}_{\mathrm{poly}}(X) \to \mathcal{T}_{\mathrm{poly}}(X')$ and $f^* : \mathcal{D}^{\mathrm{nor}}_{\mathrm{poly}}(X) \to \mathcal{D}^{\mathrm{nor}}_{\mathrm{poly}}(X')$. These homomorphisms extend to formal coefficients, and we get functions

$$\operatorname{MC}(f^*) : \operatorname{MC}\big(\mathcal{T}_{\mathrm{poly}}(X)[[\hbar]]^+\big) \to \operatorname{MC}\big(\mathcal{T}_{\mathrm{poly}}(X')[[\hbar]]^+\big)$$

etc.

One says that X is a *$\mathcal{D}$-affine variety* if $\mathrm{H}^q(X, \mathcal{M}) = 0$ for every quasi-coherent left $\mathcal{D}_X$-module $\mathcal{M}$ and every $q > 0$.

Theorem 1.14. *Let X be an irreducible smooth separated $\mathbb{K}$-scheme. Assume X is $\mathcal{D}$-affine. Then there is a canonical function*

$$Q : \operatorname{MC}\big(\mathcal{T}_{\mathrm{poly}}(X)[[\hbar]]^+\big) \to \operatorname{MC}\big(\mathcal{D}^{\mathrm{nor}}_{\mathrm{poly}}(X)[[\hbar]]^+\big)$$

called the quantization map. *It has the following properties:*

(i) *The function Q preserves first order terms.*
(ii) *The function Q respects étale morphisms. Namely if X' is another $\mathcal{D}$-affine scheme, with quantization map Q', and if $f : X' \to X$ is an étale morphism, then*
$$Q' \circ \operatorname{MC}(f^*) = \operatorname{MC}(f^*) \circ Q.$$
(iii) *If X is affine, then Q is bijective.*

(iv) *The function Q is characterized as follows. Choose an open covering $\boldsymbol{U} = \{U_0, \ldots, U_m\}$ of X consisting of affine open sets, each admitting an étale coordinate system. Let $\boldsymbol{\sigma}$ be the associated simplicial section of the bundle* $\operatorname{LCC} X \to X$. *Then there is a commutative diagram*

$$\begin{array}{ccc} \mathrm{MC}\big(\mathcal{T}_{\mathrm{poly}}(X)[[\hbar]]^+\big) & \xrightarrow{Q} & \mathrm{MC}\big(\mathcal{D}^{\mathrm{nor}}_{\mathrm{poly}}(X)[[\hbar]]^+\big) \\ {\scriptstyle \mathrm{MC}(\eta_{\mathcal{T}})}\downarrow & & {\scriptstyle \mathrm{MC}(\eta_{\mathcal{D}})}\downarrow \\ \mathrm{MC}\big(\Gamma(X, \mathrm{Mix}_{\boldsymbol{U}}(\mathcal{T}_{\mathrm{poly},X}))[[\hbar]]^+\big) & \xrightarrow{\mathrm{MC}(\Psi_{\boldsymbol{\sigma}})} & \mathrm{MC}\big(\Gamma(X, \mathrm{Mix}_{\boldsymbol{U}}(\mathcal{D}_{\mathrm{poly},X}))[[\hbar]]^+\big) \end{array}$$

in which the arrows $\mathrm{MC}(\Psi_{\boldsymbol{\sigma}})$ *and* $\mathrm{MC}(\eta_{\mathcal{D}})$ *are bijections. Here $\Psi_{\boldsymbol{\sigma}}$ is the* L_∞ *quasi-isomorphism from Theorem* 1.2, *and $\eta_{\mathcal{T}}, \eta_{\mathcal{D}}$ are the inclusions of DG Lie algebras.*

Let's elaborate a bit on the statement above. It says that to any MC solution $\alpha = \sum_{j=1}^{\infty} \alpha_j \hbar^j \in \mathcal{T}^1_{\mathrm{poly}}(X)[[\hbar]]^+$ there corresponds an MC solution $\beta = \sum_{j=1}^{\infty} \beta_j \hbar^j \in \mathcal{D}^{\mathrm{nor},1}_{\mathrm{poly}}(X)[[\hbar]]^+$. The element $\beta = Q(\alpha)$ is uniquely determined up to gauge equivalence by the group $\exp(\mathcal{D}^{\mathrm{nor},0}_{\mathrm{poly}}(X)[[\hbar]]^+)$. Given any local sections $f, g \in \mathcal{O}_X$ one has

$$\tfrac{1}{2}(\beta_1(f,g) - \beta_1(g,f)) = \alpha_1(f,g) \in \mathcal{O}_X. \tag{1.15}$$

The quantization map Q can be calculated (at least in theory) using the collection of sections $\boldsymbol{\sigma}$ and the universal formulas for deformation in [Ye1, Theorem 3.13].

We'll need a lemma before proving the theorem.

Lemma 1.16. *Let $f, g \in \mathcal{O}_X = \mathcal{D}^{-1}_{\mathrm{poly},X}$ be local sections.*

(1) *For any $\beta \in \mathrm{Mix}^0_{\boldsymbol{U}}(\mathcal{D}^1_{\mathrm{poly},X})$ one has*
$$[[\beta, f], g] = \beta(g,f) - \beta(f,g) \in \mathrm{Mix}^0_{\boldsymbol{U}}(\mathcal{O}_X).$$
(2) *For any $\beta \in \mathrm{Mix}^1_{\boldsymbol{U}}(\mathcal{D}^0_{\mathrm{poly},X}) \oplus \mathrm{Mix}^2_{\boldsymbol{U}}(\mathcal{D}^{-1}_{\mathrm{poly},X})$ one has $[[\beta, f], g] = 0$.*

(3) *Let $\gamma \in \mathrm{Mix}_{\boldsymbol{U}}(\mathcal{D}_{\mathrm{poly},X})^0$, and define $\beta := (\mathrm{d}_{\mathrm{mix}} + \mathrm{d}_{\mathcal{D}})(\gamma)$. Then $[[\beta, f], g] = 0$.*

Proof. (1) [Ye1, Proposition 6.3] implies that the embedding ([Ye1, (6.1)]:

$$\mathrm{Mix}_{\boldsymbol{U}}(\mathcal{D}_{\mathrm{poly},X}) \subset \bigoplus_{p,q,r} \prod_{j \in \mathbb{N}} \prod_{\boldsymbol{i} \in \boldsymbol{\Delta}^m_j} g_{\boldsymbol{i}*}\, g_{\boldsymbol{i}}^{-1} \left(\Omega^q(\boldsymbol{\Delta}^j_{\mathbb{K}}) \mathbin{\widehat{\otimes}} \big(\Omega^p_X \otimes_{\mathcal{O}_X} \mathcal{P}_X \otimes_{\mathcal{O}_X} \mathcal{D}^r_{\mathrm{poly},X}\big)\right)$$

is a DG Lie algebra homomorphism. So by continuity we might as well assume that $\beta = aD$ with $a \in \Omega^0_X = \mathcal{O}_X$ and $D \in \mathcal{D}^1_{\mathrm{poly},X}$. Moreover, since the Lie bracket of $\Omega_X \otimes_{\mathcal{O}_X} \mathcal{P}_X \otimes_{\mathcal{O}_X} \mathcal{D}_{\mathrm{poly},X}$ is Ω_X-bilinear, we may assume that $a = 1$, i.e. $\beta = D$. Now the assertion is clear from the definition of the Gerstenhaber Lie bracket, see [Ko, Section 3.4.2].

(2) Applying the same reduction as above, but with $D \in \mathcal{D}^r_{\mathrm{poly},X}$ and $r \in \{0, -1\}$, we get $[[D, f], g] \in \mathcal{D}^{r-2}_{\mathrm{poly},X} = 0$.

(3) By part (2) it suffices to show that $[[\beta, f], g] = 0$ for $\beta := \mathrm{d}_{\mathcal{D}}(\gamma)$ and $\gamma \in \mathrm{Mix}^0_{\boldsymbol{U}}(\mathcal{D}^0_{\mathrm{poly},X})$. As explained above we may further assume that $\gamma = D \in \mathcal{D}^0_{\mathrm{poly},X}$. Now the formulas for $\mathrm{d}_{\mathcal{D}}$ and $[-,-]$ in [Ko, Section 3.4.2] imply that $[[\mathrm{d}_{\mathcal{D}}(D), f], g] = 0$. ∎

Proof of Theorem 1.14. Step 1. Take an open covering $\boldsymbol{U}$ as in property (iv). Since the sheaves $\mathcal{D}^{\mathrm{nor},p}_{\mathrm{poly},X}$ are quasi-coherent left $\mathcal{D}_X$-modules, it follows that $\mathrm{H}^q(X, \mathcal{D}^{\mathrm{nor},p}_{\mathrm{poly},X}) = 0$ for all p and all $q > 0$. Therefore $\Gamma(X, \mathcal{D}^{\mathrm{nor}}_{\mathrm{poly},X}) = \mathrm{R}\Gamma(X, \mathcal{D}^{\mathrm{nor}}_{\mathrm{poly},X})$ in the derived category $\mathsf{D}(\mathsf{Mod}\,\mathbb{K})$. Now by [Ye1, Theorem 3.12] the inclusion $\mathcal{D}^{\mathrm{nor}}_{\mathrm{poly},X} \to \mathcal{D}_{\mathrm{poly},X}$ is a quasi-isomorphism, and by [Ye1, Theorem 6.2(1)] the inclusion $\mathcal{D}_{\mathrm{poly},X} \to \mathrm{Mix}_{\boldsymbol{U}}(\mathcal{D}_{\mathrm{poly},X})$ is a quasi-isomorphism. According to [Ye1, Theorem 6.2(2)] we have $\Gamma\big(X, \mathrm{Mix}_{\boldsymbol{U}}(\mathcal{D}_{\mathrm{poly},X})\big) = \mathrm{R}\Gamma\big(X, \mathrm{Mix}_{\boldsymbol{U}}(\mathcal{D}_{\mathrm{poly},X})\big)$. The conclusion is that

$$\mathcal{D}^{\mathrm{nor}}_{\mathrm{poly}}(X) = \Gamma(X, \mathcal{D}^{\mathrm{nor}}_{\mathrm{poly},X}) \to \Gamma\big(X, \mathrm{Mix}_{\boldsymbol{U}}(\mathcal{D}_{\mathrm{poly},X})\big) \tag{1.17}$$

is a quasi-isomorphism of complexes of $\mathbb{K}$-modules. But in view of [Ye1, Proposition 6.3], this is in fact a quasi-isomorphism of DG Lie algebras.

From (1.17) we deduce that

$$\eta_{\mathcal{D}} : \mathcal{D}^{\mathrm{nor}}_{\mathrm{poly}}(X)[[\hbar]]^+ \to \Gamma\big(X, \mathrm{Mix}_{\boldsymbol{U}}(\mathcal{D}_{\mathrm{poly},X})\big)[[\hbar]]^+$$

is a quasi-isomorphism of DG Lie algebras. Using [Ye1, Corollary 3.10] we see that $\mathrm{MC}(\eta_{\mathcal{D}})$ is bijective. Therefore the diagram in property (iv) defines Q uniquely.

According to Corollary 1.13, the arrow marked $\mathrm{MC}(\Psi_{\boldsymbol{\sigma}})$ is a bijection. So we have estanlished property (iv), except for the independence of the open covering.

Step 2. The left vertical arrow comes from the DG Lie algebra homomorphism

$$\eta_{\mathcal{T}} : \mathcal{T}_{\mathrm{poly}}(X)[[\hbar]]^+ \to \Gamma\big(X, \mathrm{Mix}_{\boldsymbol{U}}(\mathcal{T}_{\mathrm{poly},X})\big)[[\hbar]]^+,$$

which is a quasi-isomorphism when $\mathrm{H}^q(X, \mathcal{T}^p_{\mathrm{poly},X}) = 0$ for all p and all $q > 0$. So in case X is affine, the quantization map Q is bijective. This establishes property (iii).

Step 3. Now suppose $\boldsymbol{U}' = \{U'_0, \ldots, U'_{m'}\}$ is another such affine open covering of X, with sections $\sigma'_i : U'_i \to \mathrm{LCC}\, X$. Without loss of generality we may assume that $m' \geq m$, and that $U'_i = U_i$ and $\sigma'_i = \sigma_i$ for all $i \leq m$. There is a morphism of simplicial schemes $f : \boldsymbol{U} \to \boldsymbol{U}'$, that is an open and closed embedding. Correspondingly there is a commutative diagram

$$\begin{tikzcd}
\mathrm{MC}\big(\mathcal{T}_{\mathrm{poly}}(X)[[\hbar]]^+\big) \arrow[r, "Q"] \arrow[d, "\mathrm{MC}(\eta_{\mathcal{T}})"'] & \mathrm{MC}\big(\mathcal{D}^{\mathrm{nor}}_{\mathrm{poly}}(X)[[\hbar]]^+\big) \arrow[d, "\mathrm{MC}(\eta_{\mathcal{D}})"'] \\
\mathrm{MC}\big(\Gamma(X, \mathrm{Mix}_{\boldsymbol{U}'}(\mathcal{T}_{\mathrm{poly},X}))[[\hbar]]^+\big) \arrow[r, "\mathrm{MC}(\Psi_{\boldsymbol{\sigma}'})"] \arrow[d, "\mathrm{MC}(f^*)"'] & \mathrm{MC}\big(\Gamma(X, \mathrm{Mix}_{\boldsymbol{U}'}(\mathcal{D}_{\mathrm{poly},X}))[[\hbar]]^+\big) \arrow[d, "\mathrm{MC}(f^*)"'] \\
\mathrm{MC}\big(\Gamma(X, \mathrm{Mix}_{\boldsymbol{U}}(\mathcal{T}_{\mathrm{poly},X}))[[\hbar]]^+\big) \arrow[r, "\mathrm{MC}(\Psi_{\boldsymbol{\sigma}})"] & \mathrm{MC}\big(\Gamma(X, \mathrm{Mix}_{\boldsymbol{U}}(\mathcal{D}_{\mathrm{poly},X}))[[\hbar]]^+\big),
\end{tikzcd}$$

where the vertical arrows on the right are bijections. We conclude that Q is independent of $\boldsymbol{U}$ and $\boldsymbol{\sigma}$. This concludes the proof of property (iv).

Step 4. Suppose $f : X' \to X$ is an étale morphism. Then we can choose an affine open covering $\boldsymbol{U}'$ of X' that refines $\boldsymbol{U}$ in the obvious sense. Each of the open sets U'_i inherits an étale coordinate system, and hence a section $\sigma'_i : U'_i \to \mathrm{LCC}\, X'$. We

get a commutative diagram

$$\begin{array}{ccc} \mathrm{MC}\big(\Gamma(X, \mathrm{Mix}_{\boldsymbol{U}}(\mathcal{T}_{\mathrm{poly},X}))[[\hbar]]^+\big) & \xrightarrow{\mathrm{MC}(\Psi_{\boldsymbol{\sigma}})} & \mathrm{MC}\big(\Gamma(X, \mathrm{Mix}_{\boldsymbol{U}}(\mathcal{D}_{\mathrm{poly},X}))[[\hbar]]^+\big) \\ {\scriptstyle \mathrm{MC}(f^*)}\downarrow & & {\scriptstyle \mathrm{MC}(f^*)}\downarrow \\ \mathrm{MC}\big(\Gamma(X', \mathrm{Mix}_{\boldsymbol{U}'}(\mathcal{T}_{\mathrm{poly},X'}))[[\hbar]]^+\big) & \xrightarrow{\mathrm{MC}(\Psi_{\boldsymbol{\sigma}'})} & \mathrm{MC}\big(\Gamma(X', \mathrm{Mix}_{\boldsymbol{U}'}(\mathcal{D}_{\mathrm{poly},X'}))[[\hbar]]^+\big), \end{array}$$

This proves property (ii).

Step 5. Finally we must show that Q preserves first order terms, i.e. property (i). Let

$$\alpha = \sum_{j=1}^{\infty} \alpha_j \hbar^j \in \mathcal{T}_{\mathrm{poly}}(X)^1[[\hbar]]^+$$

be an MC solution, and let

$$\beta = \sum_{j=1}^{\infty} \beta_j \hbar^j \in \mathcal{D}^{\mathrm{nor}}_{\mathrm{poly}}(X)^1[[\hbar]]^+$$

be an MC solution such that $\beta = Q(\alpha)$ modulo gauge equivalence. This means that there exists some

$$\gamma = \sum_{k \geq 1} \gamma_k \hbar^k \in \Gamma(X, \mathrm{Mix}_{\boldsymbol{U}}(\mathcal{D}_{\mathrm{poly},X}))^0[[\hbar]]^+$$

such that

$$\sum_{j \geq 1} \tfrac{1}{j!} \Psi_{\boldsymbol{\sigma};j}(\alpha^j) = \exp(\mathrm{af})(\exp(\gamma))(\beta),$$

with notation as in [Ye1, Lemma 3.2]. Cf. [Ye1, Theorem 3.8]. In the first order term (i.e. the coefficient of $\hbar^1$) of this equation we have

$$\Psi_{\boldsymbol{\sigma};1}(\alpha_1) = \beta_1 - (\mathrm{d}_{\mathrm{mix}} + \mathrm{d}_{\mathcal{D}})(\gamma_1); \tag{1.18}$$

see [Ye1, equation (3.3)].

In order to apply Lemma 1.16(2), we are interested in in the component of $\Psi_{\boldsymbol{\sigma};1}(\alpha_1)$ living in the summand $\mathrm{Mix}^0_{\boldsymbol{U}}(\mathcal{D}^1_{\mathrm{poly},X})$. But this is exactly

$$\mathrm{gr}^0_{\mathrm{G}}(\Psi_{\boldsymbol{\sigma};1})(\alpha_1) \in \mathrm{gr}^0_{\mathrm{G}} \mathrm{Mix}_{\boldsymbol{U}}(\mathcal{D}^1_{\mathrm{poly},X}) = \mathrm{Mix}^0_{\boldsymbol{U}}(\mathcal{D}^1_{\mathrm{poly},X}).$$

Since according to Theorem 1.2 we have

$$\mathrm{gr}^0_{\mathrm{G}}(\Psi_{\boldsymbol{\sigma};1}) = \mathrm{gr}^0_{\mathrm{G}}(\mathrm{Mix}_{\boldsymbol{U}}(\mathcal{U}_1)),$$

it follows that the component we are interested in is

$$\mathrm{gr}^0_{\mathrm{G}}(\mathrm{Mix}_{\boldsymbol{U}}(\mathcal{U}_1))(\alpha_1) = \mathcal{U}_1(\alpha_1).$$

Now take any two local sections $f, g \in \mathcal{O}_X$. Using Lemma 1.16 we get

$$[[\Psi_{\boldsymbol{\sigma};1}(\alpha_1), f], g] = [[\mathcal{U}_1(\alpha_1), f], g] = \mathcal{U}_1(\alpha_1)(g, f) - \mathcal{U}_1(\alpha_1)(f, g) = -2\alpha_1(f, g),$$

$$[[\beta_1, f], g] = \beta_1(g, f) - \beta_1(f, g)$$

and

$$[[(\mathrm{d}_{\mathrm{mix}} + \mathrm{d}_{\mathcal{D}})(\gamma_1), f], g] = 0.$$

Combining these equations with equation (1.18) we see that equation (1.15) indeed holds. So the proof is done. $\square$

Corollary 1.19. *Let X be an irreducible smooth separated $\mathbb{K}$-scheme. Assume X is $\mathcal{D}$-affine. Then the quantization map Q of Theorem 1.14 may be interpreted as a canonical function*

$$Q : \frac{\{\text{formal Poisson structures on } X\}}{\text{gauge equivalence}} \to \frac{\{\text{deformation quantizations of } \mathcal{O}_X\}}{\text{gauge equivalence}}.$$

The quantization map Q preserves first order terms, and commutes with étale morphisms $f : X' \to X$. If X is affine then Q is bijective.

Proof. By definition the left side is $\mathrm{MC}\big(\mathcal{T}_{\mathrm{poly}}(X)[[\hbar]]^+\big)$. On the other hand, according to [Ye1, Theorem 1.13] every deformation quantization of $\mathcal{O}_X$ can be trivialized globally, and by [Ye1, Proposition 1.14] any gauge equivalence between globally trivialized deformation quantizations is a global gauge equivalence. Hence the right side is $\mathrm{MC}\big(\mathcal{D}^{\mathrm{nor}}_{\mathrm{poly}}(X)[[\hbar]]^+\big)$. $\square$

2. Miscellaneous Errors

Here is a list of minor errors in the paper [Ye1].

(1) Section 3, bottom of page 395: the formula should be

$$\mathrm{af}(\gamma)(\omega) := [\gamma, \omega] - \mathrm{d}(\gamma) = \mathrm{ad}(\gamma)(\omega) - \mathrm{d}(\gamma) \in \mathfrak{m} \mathbin{\widehat{\otimes}} \mathfrak{g}^1,$$

(2) Definition 5.2, page 411: the formula should be

$$\nabla_{\mathcal{P}} : \mathcal{P}_X \to \Omega^1_X \otimes_{\mathcal{O}_X} \mathcal{P}_X.$$

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